CIRCULAR MOTION

If a particle moves in a plane such that its distance from a fixed (or moving) point remains constant then its motion is called as circular motion with respect to that fixed or moving point.

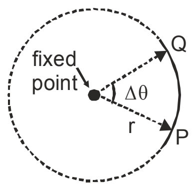
That fixed point is called the centre and the corresponding distance is called the radius of circular path.

The vector joining the centre of the circle and the particle performing circular motion, directed towards the later is called the radius vector. It has constant magnitude but variable direction.

1. KINEMATICS OF CIRCULAR MOTION

Angular Displacement

Angle traced by the position vector of a particle moving w.r.t. some fixed point is called angular displacement.



$\Delta\theta$ = angular displacement $\quad \because \quad$ angle $= \dfrac{\text{arc}}{\text{radius}} \quad \therefore \quad \Delta\theta = \dfrac{\text{arc } PQ}{r}$

Frequency (n) : Number of revolutions described by particle per second is its frequency. Its unit is revolutions per second (rps) or revolutions per minute (rpm).

Note : 1 rps = 60 rpm

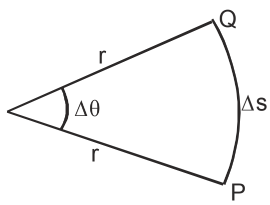
Time Period (T) : It is the time taken by particle to complete one revolution. i.e. $T = \dfrac{1}{n}$

Angular Velocity (ω) : It is defined as the rate of change of angular displacement of a moving particle, w.r.t. to time.

$$\omega = \frac{\text{angle traced}}{\text{time taken}} = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

$\Rightarrow$ Its unit is rad/s and dimensions is $[T^{-1}]$

Relation between linear and Angular velocity



Angle $(\Delta\theta) = \dfrac{\text{arc}}{\text{radius}} = \dfrac{\Delta s}{r} \quad \Rightarrow \quad \Delta s = r\Delta\theta$

$\therefore \dfrac{\Delta s}{\Delta t} = \dfrac{r\Delta\theta}{\Delta t}$ if $\Delta t \to 0$ then $\dfrac{ds}{dt} = r\dfrac{d\theta}{dt} \quad \Rightarrow \quad \boxed{v = \omega r}$

In vector form $\quad \boxed{\vec{v} = \vec{\omega} \times \vec{r}}$ (direction of $\vec{v}$ is according to right hand thumb rule)

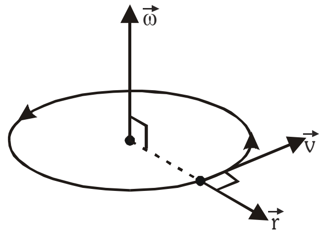
Here, $\quad \vec{v}$ = Linear velocity $\quad$ [Tangential vector]

$\vec{\omega}$ = Angular velocity $\quad$ [Axial vector]

$\vec{r}$ = Radius vector or position vector

Note : Centrifugal means away from the centre and centripetal means towards the centre.

All the three vectors $\vec{v}, \vec{\omega}$ and $\vec{r}$ are mutually perpendicular to each other.

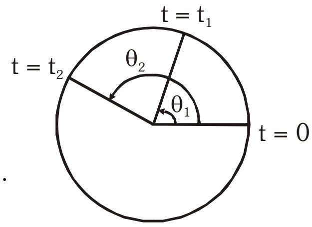


Here, $\vec{v} \perp \vec{\omega} \perp \vec{r}$ $\quad \because \quad \vec{v} \perp \vec{\omega} \quad \therefore \quad \vec{v}.\vec{\omega} = 0$

$\because \quad \vec{\omega} \perp \vec{r} \quad \therefore \quad \vec{\omega}.\vec{r} = 0$

$\because \quad \vec{v} \perp \vec{r} \quad \therefore \quad \vec{v}.\vec{r} = 0$

Average Angular Velocity (ω_{av})



$$\omega_{av} = \frac{\text{total angle of rotation}}{\text{total time taken}} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t}$$

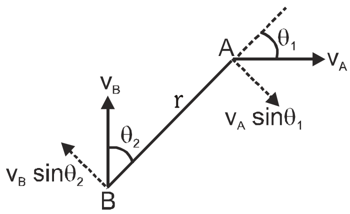
where θ_1 and θ_2 are the angular positions of the particle at instants t_1 and t_2.

Instantaneous Angular Velocity

The angular velocity at some particular instant $\vec{\omega} = \lim\limits_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\vec{\theta}}{dt}$

Relative Angular Velocity

Relative angular velocity of a particle 'A' w.r.t. an other moving particle B is the angular velocity of the position vector of A w.r.t. B. It means the rate at which the position vector of 'A' w.r.t. B rotates at that instant.



$$\omega_{AB} = \frac{(v_{AB})_\perp}{r_{AB}} = \frac{\text{relative velocity of A w.r.t. B perpendicular to line AB}}{\text{separation between A and B}}$$

Here $(v_{AB})_\perp = v_A \sin\theta_1 + v_B \sin\theta_2 \quad \therefore \quad \omega_{AB} = \frac{v_A \sin\theta_1 + v_B \sin\theta_2}{r}$

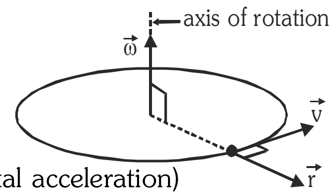
Angular Acceleration (α)

Rate of change of angular velocity is called angular acceleration. i.e. $\vec{\alpha} = \lim\limits_{\Delta t \to 0} \frac{\Delta\vec{\omega}}{\Delta t} = \frac{d\vec{\omega}}{dt}$

Average Angular Acceleration :

$$\boxed{\vec{\alpha}_{avg} = \frac{\text{change in angular velocity}}{\text{time taken}} = \frac{\Delta\vec{\omega}}{\Delta t}}$$

Relation between Angular and Linear Accelerations



Velocity $\vec{v} = \vec{\omega} \times \vec{r}$

Acceleration $\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}(\vec{\omega} \times \vec{r}) = \frac{d\vec{\omega}}{dt} \times \vec{r} + \vec{\omega} \times \frac{d\vec{r}}{dt}$

$\Rightarrow \vec{a} = \vec{\alpha} \times \vec{r} + \vec{\omega} \times \vec{v} = \vec{a} = \vec{a}_T + \vec{a}_C$

($\vec{a}_T = \vec{\alpha} \times \vec{r}$ is tangential acceleration & $\vec{a}_C = \vec{\omega} \times \vec{v}$ is centripetal acceleration)

$\vec{a} = \vec{a}_T + \vec{a}_C$ ($\vec{a}_T$ and $\vec{a}_C$ are the two component of net linear acceleration)

As $\vec{a}_T \perp \vec{a}_C$ so $|\vec{a}| = \sqrt{a_T^2 + a_C^2}$

Tangential Acceleration

$\vec{a}_T = \vec{\alpha} \times \vec{r}$, its direction is parallel (or antiparallel) to velocity. $\vec{v} = \vec{\omega} \times \vec{r}$ as $\vec{\omega}$ and $\vec{\alpha}$ **both are parallel (or antiparallel) and along the axis.**

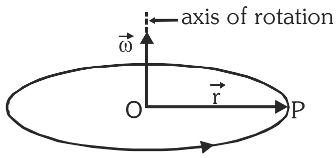
Magnitude of tangential acceleration in case of circular motion :

$$a_T = \alpha\, r \sin 90° = \alpha r \quad (\vec{\alpha} \text{ is axial, } \vec{r} \text{ is radial so that } \vec{\alpha} \perp \vec{r})$$

As $\vec{a}_T$ is along the direction of motion (in the direction of $\vec{v}$ or opposite to $\vec{v}$) so $\vec{a}_T$ is responsible for change in speed of the particle. Its magnitude is the rate of change of speed of the particle.

Note : If a particle is moving on a circular path with constant speed then tangential acceleration is zero.

Centripetal acceleration



$$\vec{a}_C = \vec{\omega} \times \vec{v} = \vec{\omega} \times (\vec{\omega} \times \vec{r}) \quad (\because \vec{v} = \vec{\omega} \times \vec{r})$$

Let $\vec{r}$ be in $\hat{i}$ direction and $\vec{\omega}$ be in $\hat{j}$ direction then the direction of

$\vec{a}_C$ is along $\hat{j} \times (\hat{j} \times \hat{i}) = \hat{j} \times (-\hat{k}) = -\hat{i}$, opposite to the direction of $\vec{r}$ i.e., from P to O and it is centripetal in direction. Magnitude of centripetal

acceleration, $a_C = \omega v = \dfrac{v^2}{r} = \omega^2 r$ therefore $\vec{a}_C = \dfrac{v^2}{r}(-\hat{r})$

Note : Centripetal acceleration is always perpendicular to the velocity at each point.

2. UNIFORM AND NON-UNIFORM CIRCULAR MOTION

2.1 Uniform Circular Motion

If a particle moves with a constant speed in a circle, the motion is called uniform circular motion. In uniform circular motion a resultant non-zero force acts on the particle. The acceleration is due to the change in direction of the velocity vector. In uniform circular motion tangential acceleration (a_T) is zero. The acceleration of the particle is towards the centre and its magnitude is $\dfrac{v^2}{r}$. Here, v is the speed of the particle and r the radius of the circle.

The direction of the resultant force F is therefore, towards the centre and its magnitude is $F = \dfrac{mv^2}{r} = mr\omega^2$ (as $v = r\omega$)

Here, ω is the angular speed of the particle. This force F is called the **centripetal force**. Thus, a centripetal force of magnitude $\dfrac{mv^2}{r}$ is needed to keep the particle moving in a circle with constant speed. This force is provided by some external agent such as friction, magnetic force, coulomb force, gravitational force, tension. etc.

In this motion :

- Speed = constant
- $|\text{Velocity}|$ = constant
- Velocity $\neq$ constant (becuase its direction continuously changes)
- K.E. $= \dfrac{1}{2}mv^2$ = constant
- $\vec{\omega}$ = constant (because magnititude and direction, both are constants)
- $a_T = 0$ $\quad \left[\because a_T = \dfrac{dv}{dt} = \dfrac{d(\text{constant})}{dt} = 0\right]$

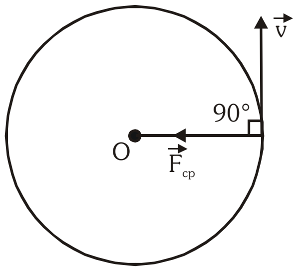
- $\alpha = 0$ $\left[\because \alpha = \dfrac{a_T}{r} = \dfrac{0}{r} = 0\right]$ or $\left[\alpha = \dfrac{d\omega}{dt} = \dfrac{d}{dt}(\text{constant}) = 0\right]$
- $|\vec{a}| = |\vec{a}_{cp}| = \omega v = \omega^2 r = \dfrac{v^2}{r} = \text{constant}$
- $\vec{a} = \vec{a}_{cp} \neq \text{constant}$

(because the direction of a_{cp} is toward the centre of circle which changes as the particle revolves)

- Total work done $= \vec{F}_{net}.\vec{s}$

$= \vec{F}_{cp}.\vec{s}$ $\left[\because \vec{F}_{net} = \vec{F}_{cp}\right]$

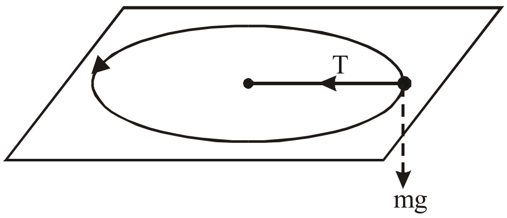
$= F_{cp}\ s\cos 90° = 0$



- Power $= \dfrac{\text{Work}}{\text{time}} = \dfrac{0}{t} = 0$ or Power $= \vec{F}_{net}.\vec{v} = \vec{F}_{cp}.\vec{v} = F_{cp} v\cos 90° = 0$
- Uniform circular motion is usually executed in a horizontal plane.

Example :

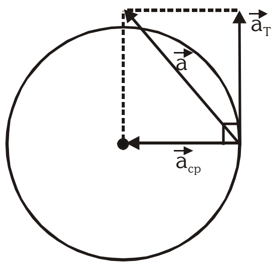
A particle of mass 'm' is tied at one end of a string of length 'r' and it is made to revolve along a circular path in a horizontal plane with a constant speed means a (uniform circular motion) In this condition the required centripetal force is provided by the tension in the string.



$T = F_{cp}$ So, $\boxed{T = \dfrac{mv^2}{r}}$

2.2 Non-Uniform Circular Motion :

If a particle moves with variable speed in a circle, then the motion is called non-uniform circular motion.



In this motion :

Accleration (a) has two components :-

$\vec{a}_{cp}$ = responsible for change in direction only.

$\vec{a}_T$ = responsible for change in speed only.

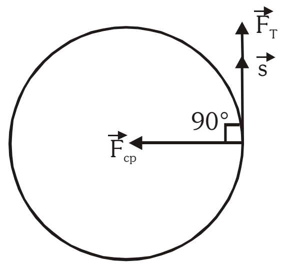
Hence due to a_T

speed = | velocity | is variable,

As speed variable hence given physical quantities are also variable.

- K.E. $= \dfrac{1}{2}mv^2$
- $|\vec{\omega}| = \dfrac{v}{r}$
- $\alpha \neq 0$
- $a_T \neq 0$
- $|\vec{a}_{cp}| = \omega v = \omega^2 r = \dfrac{v^2}{r}$
- $\vec{a} = \vec{a}_T + \vec{a}_{cp}$
- $|\vec{a}| = \sqrt{a_{T^2} + a_{cp^2}} = \sqrt{(\alpha r)^2 + \left(\dfrac{v^2}{r}\right)^2}$
- $\vec{F} = \vec{F}_T + \vec{F}_{cp}$
- $|\vec{F}| = \sqrt{F_T^2 + F_{cp}^2}$

- Work done by centripetal force will be zero but work done by tangential force is not zero.



- Total work done

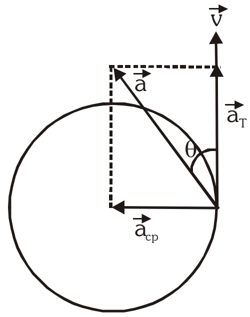
$$W = \vec{F}_T.\vec{s} = F_T \text{ s cos } 0° = F_T s$$

$\Rightarrow$ (where s is the distance travelled by the particle)

- Power $= \dfrac{\text{work}}{\text{time}} = \dfrac{\vec{F}_T.\vec{s}}{t} = \vec{F}_T.\vec{v} = F_T v \cos 0° = F_T v$

- Angle between velocity and acceleration is given by :

$$\boxed{\tan\theta = \frac{a_{cp}}{a_T} = \frac{F_{cp}}{F_T}}$$



- **Example**

Circular motion in vertical plane is an example of non-uniform circular motion.

2.3 Equations of circular motion

Translatory / Linear Motion	**Rotational Motion**
• Initial velocity (u)	Initial angular velocity (ω_0)
• Final velocity (v)	Final angular velocity (ω)
• Displacement (s)	Angular displacement (θ)
• Acceleration (a)	Angular Acceleration (α)
If a = constant, then $v = u + at$ $s = ut + \frac{1}{2}at^2$ $v^2 = u^2 + 2as$ $s_{n^{th}} = u + \frac{a}{2}(2n-1)$ $s = \left(\frac{u+v}{2}\right)t$	If α = constant, then $\omega = \omega_0 + \alpha t$ $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$ $\omega^2 = \omega_0^2 + 2\alpha\theta$ $\theta_{n^{th}} = \omega_0 + \frac{\alpha}{2}(2n-1)$ $\theta = \left(\frac{\omega_0+\omega}{2}\right)t$

GOLDEN KEY POINTS

- Small angular displacement $d\vec{\theta}$ is a vector quantity, but large angular displacement θ is not a vector quantity.

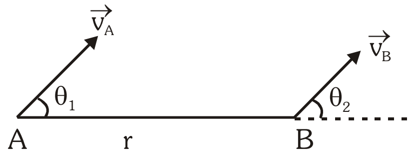
$d\vec{\theta}_1 + d\vec{\theta}_2 = d\vec{\theta}_2 + d\vec{\theta}_1$ But $\vec{\theta}_1 + \vec{\theta}_2 \neq \vec{\theta}_2 + \vec{\theta}_1$

- Direction of angular displacement is perpendicular to the plane of rotation and is given by right hand thumb rule.
- Angular displacement is dimensionless and its S.I. unit is radian while other units are degree and revolution.

2π radian = 360° = 1 revolution

- Instantaneous angular velocity is an axial vector quantity.
- Direction of angular velocity is same as that of angular displacement i.e. perpendicular to the plane of rotation and along the axis according to right hand screw rule or right hand thumb rule.

- If particles A and B are moving with a velocity $\vec{v}_A$ and $\vec{v}_B$ and are separated by a distance r at a given instant then



(i) $\frac{dr}{dt} = v_B\cos\theta_2 - v_A\cos\theta_1$ (ii) $\frac{d\theta_{BA}}{dt} = \omega_{BA} = \frac{v_B\sin\theta_2 - v_A\sin\theta_1}{r}$

- Angular acceleration is an axial vector quantity. It's direction is along the axis according to the right hand thumb rule or right hand screw rule.
- **Important difference between projectile motion and uniform circular motion :**

  In projectile motion, both the magnitude and the direction of acceleration (g) remain constant, while in uniform circular motion the magnitude remains constant but the direction continuously changes.

Illustrations

Illustration 1.

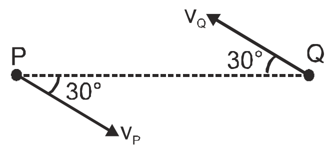
A particle revolving in a circular path completes first one third of the circumference in 2 s, while next one third in 1s. Calculate its average angular velocity.

Solution

$\theta_1 = \frac{2\pi}{3}$ and $\theta_2 = \frac{2\pi}{3}$ total time T = 2 + 1 = 3 s $\therefore < \omega_{av} > = \frac{\theta_1+\theta_2}{T} = \frac{\frac{2\pi}{3}+\frac{2\pi}{3}}{3} = \frac{\frac{4\pi}{3}}{3} = \frac{4\pi}{9}$ rad/s

Illustration 2.

Two moving particles P & Q are 10 m apart at any instant. Velocity of P is 8 m/s and that of Q is 6 m/s at 30° angle with the line joining P & Q. Calculate the angular velocity of P w.r.t. Q

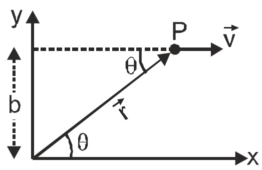


Solution

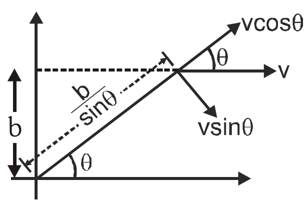
$\omega_{PQ} = \frac{8\sin 30^\circ - (-6\sin 30^\circ)}{10} = 0.7$ rad/s.

Illustration 3.

A particle is moving parallel to x–axis as shown in fig. such that the y component of its position vector is constant at all instants and is equal to 'b'. Find the angular velocity of the particle about the origin when its radius vector makes an angle θ with the x-axis.



Solution



$\therefore \quad \omega_{PO} = \frac{v\sin\theta}{\frac{b}{\sin\theta}} = \frac{v}{b}\sin^2\theta$

Illustration 4.

The angular velocity of a particle is given by $\omega = 1.5t - 3t^2 + 2$. (where t is in seconds). Find the instant when its angular acceleration becomes zero.

Solution

$\alpha = \frac{d\omega}{dt} = 1.5 - 6t = 0 \Rightarrow t = 0.25$ s.

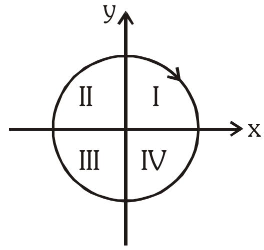
Illustration 5.

A disc starts from rest and gains an angular acceleration given by $\alpha = 3t - t^2$ (where t is in seconds) upon the application of a torque. Calculate its angular velocity after 2 s.

Solution

$$\alpha = \frac{d\omega}{dt} = 3t - t^2 \Rightarrow \int_0^{\omega} d\omega = \int_0^{t}(3t - t^2)dt \Rightarrow \omega = \frac{3t^2}{2} - \frac{t^3}{3} \Rightarrow \text{at } t = 2 \text{ s}, \omega = \frac{10}{3} \text{ rad/s}$$

Illustration 6.



A particle is moving in clockwise direction in a circular path as shown in figure.

The instantaneous velocity of particle at a certain instant is $\vec{v} = (3\hat{i} + 3\hat{j})$ m/s.

Then in which quardant does the particle lie at that instant ? Explain your answer.

Solution

II quardant. According to following figure x & y components of velocity are positive when the particle is in II quardant.

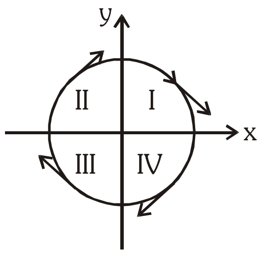


Illustration 7.

A particle is performing circular motion of radius 1 m. Its speed is $v = (2t^2)$ m/s. What will be the magnitude of its acceleration at t = 1s ?

Solution

Tangential acceleration $= a_T = \frac{dv}{dt} = 4t$, at t = 1s, $a_T = 4 \text{ m/s}^2$

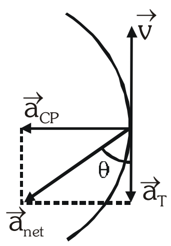
Centripetal acceleration $a_c = \frac{v^2}{r} = \frac{4t^4}{1} = 4t^4$, at t = 1 s, $a_c = 4 \text{ m/s}^2$

Net acceleration (a) $= \sqrt{a_T^2 + a_c^2} = \sqrt{4^2 + 4^2} = 4\sqrt{2} \text{ m/s}^2$.

Illustration 8.

A cyclist is riding with a speed of 18 km/h. As he approaches a circular turn on the road of radius $25\sqrt{2}$ m, he applies brakes which reduces his speed at a constant rate of 0.5 m/s every second. Determine the magnitude and direction of his net acceleration on the circular turn.

Solution



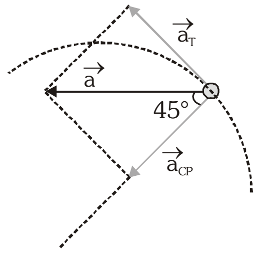
$$v = 18 \times \frac{5}{18} = 5 \text{ m/s and } a_{cp} = \frac{v^2}{R} = \frac{25}{25\sqrt{2}} = \frac{1}{\sqrt{2}} \text{m/s}^2, \; a_T = -\frac{dv}{dt} = -\frac{1}{2} \text{m/s}^2$$

$$a_{net} = \sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{2}\right)^2} = \frac{\sqrt{3}}{2} = 0.86 \text{ m/s}^2, \quad \tan\theta = \frac{a_{CP}}{a_T} = \frac{1/\sqrt{2}}{1/2} = \frac{2}{\sqrt{2}} = \sqrt{2}$$

$\theta = \tan^{-1}\left(\sqrt{2}\right)$ from tangential direction

Illustration 9.

A particle is moving in a circular orbit with a constant tangential acceleration starting from rest. After 2 s of the beginning of its motion, angle between the acceleration vector and the radius becomes 45°. What is the angular acceleration of the particle ?



Solution

In the adjoining figure the total acceleration vector $\vec{a}$ and its components-the tangential acceleration $\vec{a}_T$ and normal acceleration $\vec{a}_{CP}$ are shown. These two components are always mutually perpendicular to each other and act along the tangent to the circle and radius respectively. Therefore, if the total acceleration vector makes an angle of 45° with the radius, both the tangential and the normal components must be equal in magnitude.

$$a_T = a_{cp} \Rightarrow \alpha R = \omega^2 R \Rightarrow \alpha = \omega^2 \quad \text{...(i)}$$

Since angular acceleration is uniform, we have $\omega = \omega_o + \alpha t$

Substituting $\omega_0=0$ and $t = 2$ s, we have $\omega = 2\alpha$ ...(ii)

From equations (i) and (ii), we have $\alpha = 0.25$ rad/s^2

BEGINNER'S BOX-1

1. If angular velocity of a particle depends on the angle rotated θ as $\omega = \theta^2 + 2\theta$, then its angular acceleration α at $\theta = 1$ rad is :

(A) 8 rad/s^2 (B) 10 rad/s^2 (C) 12 rad/s^2 (D) None of these

2. The second's hand of a watch has 6 cm length. The speed of its tip and magnitude of difference in velocities of its at any two perpendicular positions will be respectively :

(A) 2π & 0 mm/s (B) $2\sqrt{2}\,\pi$ & 4.44 mm/s

(C) $2\sqrt{2}\,\pi$ & 2π mm/s (D) 2π & $2\sqrt{2}\pi$ mm/s

3. A particle is moving on a circular path of radius 6 m. Its linear speed is $v = 2t$, here t is time in second and v is in m/s. Calculate its centripetal acceleration at t = 3 s.

4. Two particles move in concentric circles of radii r_1 and r_2 such that they maintain a straight line through the centre. Find the ratio of their angular velocities.

5. If the radii of circular paths of two particles are in the ratio of 1 : 2, then in order to have same centripetal acceleration, their speeds should be in the ratio of :

(A) 1 : 4 (B) 4 : 1 (C) $1:\sqrt{2}$ (D) $\sqrt{2}:1$

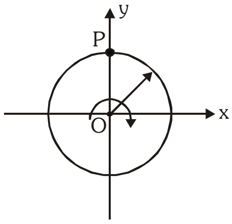
6. A stone tied to the end of a 80 cm long string is whirled in a horizontal circle with a constant speed. If the stone makes 14 revolutions in 25 s, the magnitude of its acceleration is :

(A) 20 m/s^2 (B) 12 m/s^2 (C) 9.9 m/s^2 (D) 8 m/s^2

7. For a body in a circular motion with a constant angular velocity, the magnitude of the average acceleration over a period of half a revolution is.... times the magnitude of its instantaneous acceleration.

(A) $\frac{2}{\pi}$ (B) $\frac{\pi}{2}$ (C) π (D) 2

8. A ring rotates about z axis as shown in figure. The plane of rotation is xy. At a certain instant the acceleration of a particle P (shown in figure) on the ring is $\left(6\hat{i}-8\hat{j}\right) m/s^2$. Find the angular acceleration of the ring and its angular velocity at that instant. Radius of the ring is 2 m.



3. DYNAMICS OF CIRCULAR MOTION

3.1 Circular Turning on Roads

When vehicles go through turnings, they travel along a nearly circular arc. There must be some force which provides the required centripetal acceleration. If the vehicles travel in a horizontal circular path, this resultant force is also horizontal. The necessary centripetal force is being provided to the vehicles by the following three ways :

• By friction only. • By banking of roads only. • By friction and banking of roads both.

In real life the necessary centripetal force is provided by friction and banking of roads both.

• **By Friction only**

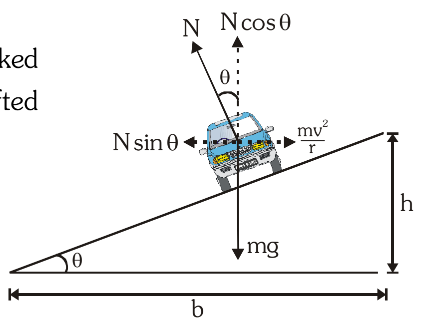
Suppose a car of mass m is moving with a speed v in a horizontal circular arc of radius r. In this case, the necessary centripetal force will be provided to the car by the force of friction f acting towards centre of the circular path.

Thus, $f = \frac{mv^2}{r}$ $\qquad \because f_{max} = \mu N = \mu mg$

Therefore, for a safe turn without skidding $\frac{mv^2}{r} \le f_{max} \Rightarrow \frac{mv^2}{r} \le \mu mg \Rightarrow v \le \sqrt{\mu rg}$.

• **By Banking of Roads only**

Friction is not always reliable at turns particularly when high speeds and sharp turns are involved. To avoid dependence on friction, the roads are banked at the turn in the sense that the outer part of the road is some what lifted compared to the inner part.



$N\sin\theta = \frac{mv^2}{r}$ and $N\cos\theta = mg$

$\Rightarrow \tan\theta = \frac{v^2}{rg} \Rightarrow v = \sqrt{rg\tan\theta}$ $\qquad \because \tan\theta = \frac{h}{b}$

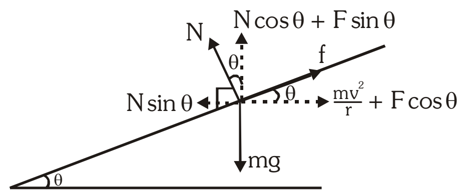
Note : $\tan\theta = \frac{v^2}{rg} = \frac{h}{b}$

- **Friction and Banking of Road both**

  If a vehicle is moving on a circular road which is rough and banked also, then three forces may act on the vehicle, of these the first force, i.e., weight (mg) is fixed both in magnitude and direction. The direction of second force, i.e., normal reaction N is also fixed (perpendicular to road) while the direction of the third force, i.e., friction f can be either inwards or outwards while its magnitude can be varied upto a maximum limit ($f_{max} = \mu N$).

  So, direction and the magnitude of friction f are so adjusted that the resultant of the three forces mentioned above is $\frac{mv^2}{r}$ towards the centre.

(a) If speed of the vehicle is small then friction acts outwards.



In this case, $N \cos\theta + f \sin\theta = mg$ ...(i)

and $N \sin\theta - f \cos\theta = \frac{mv^2}{R}$ ...(ii)

For minimum speed $f = \mu N$ so by dividing Eqn. (1) by Eqn. (2)

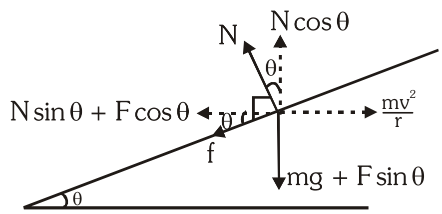
$$\frac{N\cos\theta - \mu N\sin\theta}{N\sin\theta - \mu N\cos\theta} = \frac{mg}{mv_{min}^2/R}$$

Therefore $v_{min} = \sqrt{Rg\left(\frac{\tan\theta - \mu}{1 + \mu\tan\theta}\right)}$

If we assume $\mu = \tan\phi$, then

$$v_{min} = \sqrt{Rg\left(\frac{\tan\theta - \tan\phi}{1 + \tan\phi\tan\theta}\right)} = \sqrt{Rg\tan(\theta - \phi)}$$

(b) If speed of vehicle is high then friction force act inwards.



in this case for maximum speed

$N \cos\theta - \mu N \sin\theta = mg$

and $N \sin\theta + \mu N \cos\theta = \frac{mv_{max}^2}{R}$

which gives $v_{max} = \sqrt{Rg\left(\frac{\tan\theta + \mu}{1 - \mu\tan\theta}\right)} = \sqrt{Rg\tan(\theta + \phi)}$

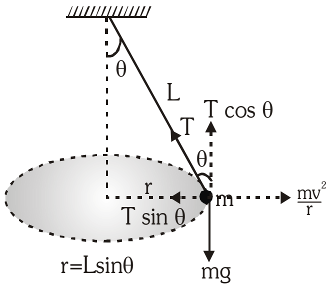
Hence for successful turning on a rough banked road, velocity of vehicle must satisfy following relation

$$\sqrt{Rg\tan(\theta - \phi)} \le v \le \sqrt{Rg\tan(\theta + \phi)}$$

where θ = banking angle and $\phi = \tan^{-1}(\mu)$.

3.2 Conical Pendulum

If a small particle of mass m tied to a string is whirled along a horizontal circle, as shown in figure then the arrangement is called a 'conical pendulum'. In case of conical pendulum the vertical component of tension balances the weight while its horizontal component provides the necessary centripetal force. Thus,



$$T\sin\theta = \frac{mv^2}{r} \quad \text{and} \quad T\cos\theta = mg \Rightarrow v = \sqrt{rg\tan\theta}$$

$\therefore$ Angular speed $\quad \omega = \frac{v}{r} = \sqrt{\frac{g\tan\theta}{r}}$

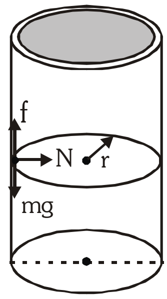
So, the time period of pendulum is $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{r}{g\tan\theta}} = 2\pi\sqrt{\frac{L\cos\theta}{g}}$

3.3 'Death Well' or Rotor

In case of 'death well' a person drives a motorcycle on the vertical surface of a large wooden well while in case of a rotor a person hangs resting against the wall without any support from the bottom at a certain angular speed of rotor. In death well walls are at rest and person revolves while in case of rotor person is at rest and the walls rotate.

In both cases friction balances the weight of person while reaction provides the centripetal force for circular motion, i.e.,

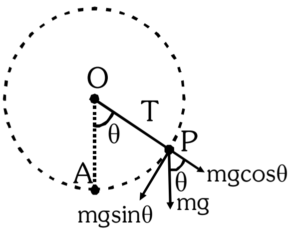
$$f = mg \text{ and } N = \frac{mv^2}{r} = mr\omega^2$$



Death well

4. VERTICAL CIRCULAR MOTION

Suppose a particle of mass m is attached to a light inextensible string of length R. The particle is moving in a vertical circle of radius R about a fixed point O. It is imparted a velocity u in the horizontal direction at lowest point A. Let v be its velocity at point P of the circle as shown in the figure.



When a particle is whirled in a vertical circle then three cases are possible -

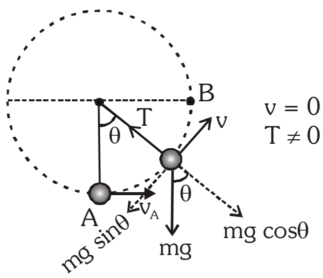
Case I : Particle oscillates in lower half circle.

Case II : Particle moves to upper half circle but not able to complete loop.

Case III : Particle completes loop.

Condition of Oscillation $\left(0 < u \le \sqrt{2gR}\right)$

The particle will oscillate if velocity of the particle becomes zero but tension in the string is not zero. (In lower half circle (A to B))



Here, $T - mg\cos\theta = \frac{mv_A^2}{R}$

$$T = \frac{mv_A^2}{R} + mg\cos\theta$$

In the lower part of circle when velocity become zero and tension is non zero means when v = 0, but $T \neq 0$

So, to make the particle ossillate in lower half cycle, maximum possible velocity at A can be. given by

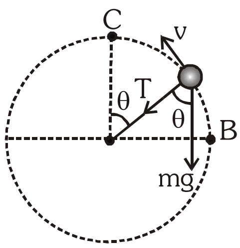
$\frac{1}{2}mv_A^2 + 0 = mgR + 0$ (by COME between A and B)

$v_A = \sqrt{2gR}$ ....(i)

Thus, for $0 < u \leq \sqrt{2gR}$, particle oscillates in lower half of the circle ($0^0 < \theta \leq 90^0$)

This situation is shown in the figure. $0 < u \leq \sqrt{2gR}$ or $0° < \theta \leq 90°$

Condition of Leaving the Circle : $\left(\sqrt{2gR} < u < \sqrt{5gR}\right)$



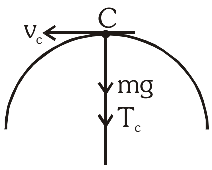
In upper half cycle (B to C)

Here, $T + mg\cos\theta = \frac{mv^2}{R}$

$T = \left(\frac{mv^2}{R} - mg\cos\theta\right)$ .....(ii)

In this part of circle tension force can be zero without having zero velocity mean when T = 0, v ≠ 0

form equation (ii) it is clear that tension decreses if velocity decreases. So to complete the loop tension force should not be zero, in between B to C. Tension will be minimum at C i.e., $T_c \geq 0$ is the required condition.



At Top $T_c + mg = \frac{mv_c^2}{R}$

if $T_c = 0$

Then $mg = \frac{mv_c^2}{R}$

$v_c^2 = gR \Rightarrow v_c = \sqrt{gR}$

By COME (Between A and C)

$\frac{1}{2}mv_A^2 + 0 = \frac{1}{2}mv_c^2 + mg(2R)$

$v_A^2 = v_c^2 + 4gR \Rightarrow v_A^2 = 5gR \Rightarrow v_A = \sqrt{5gR}$

Therefore, if $\sqrt{2gR} < u < \sqrt{5gR}$, the particle leaves the circle.

Note : After leaving the circle, the particle will follow a parabolic path.

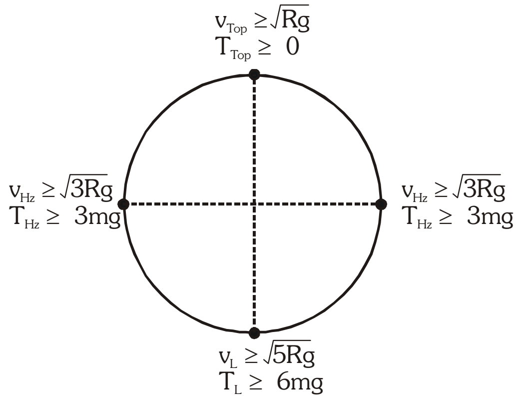
Condition of Looping the Loop $\left(u \geq \sqrt{5gR}\right)$

The particle will complete the circle if the string does not slack even at the highest point $(\theta = \pi)$. Thus, tension in the string should be greater than or equal to zero $(T \geq 0)$ at $\theta=\pi$. In critical case substituting T=0

Thus, if $u \geq \sqrt{5gR}$, the particle will complete the circle.

Note : In case of light rod tension at top most point can never be zero so velocity will become zero.

$\therefore$ For completing the loop $\quad v_L \geq \sqrt{4gR}$



Illustrations

Illustration 10.

Find the maximum speed at which a car can turn round a curve of 30 m radius on a levelled road if the coefficient of friction between the tyres and the road is 0.4 [acceleration due to gravity = 10 m/s^2]

Solution

Here centripetal force is provided by friction so

$$\frac{mv^2}{r} \leq \mu mg \Rightarrow v_{max} = \sqrt{\mu rg} = \sqrt{120} \approx 11 \text{ m/s}$$

Illustration 11.

For traffic moving at 60 km/h, if the radius of the curve is 0.1 km, what is the correct banking angle of the road ? (g = 10 m/s^2)

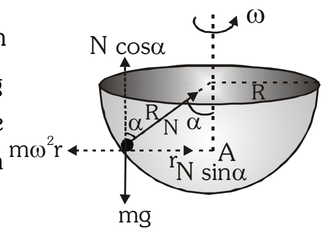
Solution

In case of banking $\tan\theta = \frac{v^2}{rg}$. Here v= 60 km/h = 60 $\times \frac{5}{18}$ m/s = $\frac{50}{3}$ m/s r = 0.1 km = 100 m

So $\tan\theta = \frac{50/3 \times 50/3}{100 \times 10} = \frac{5}{18} \Rightarrow \theta = \tan^{-1}\left(\frac{5}{18}\right)$.

Illustration 12.

A hemispherical bowl of radius R is rotating about its axis of symmetry which is kept vertical. A small ball kept in the bowl rotates with the bowl without slipping on its surface. If the surface of the bowl is smooth and the angle made by the radius through the ball with the vertical is α. Find the angular speed at which the bowl is rotating.



Solution

N cosα = mg .....(1)

N sinα = mrω^2 .....(2)

r = Rsinα .....(3)

From equations (2) & (3)

Nsinα = mω^{2}Rsinα

N=mRω^2 .....(4)

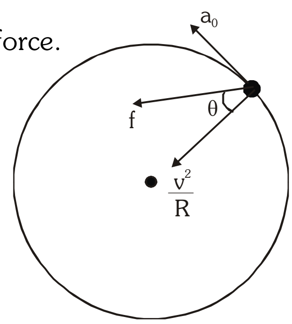
$\Rightarrow (mR\omega^2)\cos\alpha = mg \quad \Rightarrow \omega = \sqrt{\frac{g}{R\cos\alpha}}$

Illustration 13.

A car starts from rest with a constant tangential acceleration a_0 in a circular path of radius r. At time t_0, the car skids, find the value of coefficient of friction.

Solution

The tangential and centripetal acceleration is provided only by the frictional force.



Thus, $f\sin\theta = ma_0$ and $f\cos\theta = \frac{mv^2}{r} = \frac{m(a_0t_0)^2}{r}$

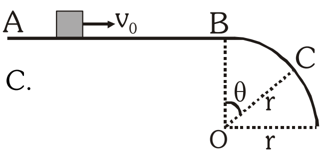
$$\Rightarrow f = m\sqrt{a_0^2 + \frac{(a_0t_0)^4}{r^2}} \Rightarrow ma_0\sqrt{1+\frac{a_0^2t_0^4}{r^2}} = f_{max}$$

(since the car skids beyound this speed)

$$\mu mg = ma_0\sqrt{1+\frac{a_0^2t_0^4}{r^2}} \Rightarrow \mu = \frac{a_0}{g}\sqrt{1+\frac{a_0^2t_0^4}{r^2}}.$$

Illustration 14.

A small block slides with a velocity $0.5\sqrt{gr}$ on a horizontal frictionless surface as shown in the figure. The block leaves the surface at point C. Calculate the angle θ shown in the figure.



Solution

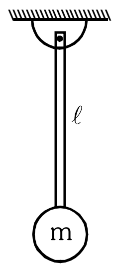
As the block leaves the surface at C so there, normal reaction = 0 $\Rightarrow mg\cos\theta = \frac{mv_C^2}{r}$

By energy conservation at points B & C, $\frac{1}{2}mv_C^2 - \frac{1}{2}mv_0^2 = mgr(1-\cos\theta)$

$$\Rightarrow \frac{1}{2}m(rg\cos\theta) - \frac{1}{2}m\left(0.5\sqrt{gr}\right)^2 = mgr(1-\cos\theta) \Rightarrow \cos\theta = \frac{3}{4} \Rightarrow \theta = \cos^{-1}\left(\frac{3}{4}\right)$$

Illustration 15.

A rigid rod of length ℓ and negligible mass has a ball of mass m attached to one end with its other end fixed, to form a pendulum as shown in figure. The pendulum is inverted, with the rod vertically up, and then released. Find the speed of the ball and the tension in the rod at the lowest point of the trajectory of ball.



Solution

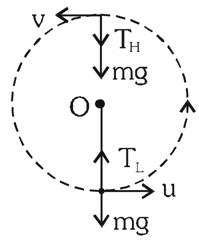
From COME : $2mg\ell = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{4g\ell} = 2\sqrt{g\ell}$

At the lowest point, laws of circular dynamics yield, $T - mg = \frac{mv^2}{\ell} \Rightarrow T = mg + \frac{m}{\ell}(4g\ell) = 5mg$.

Illustration 16.

A particle of mass m tied to a string of length ℓ and given a circular motion in the vertical plane. If it performs the complete loop motion then prove that difference in tensions at the lowest and the highest point is 6 mg.

Solution



Let the speeds at the lowest and highest points be u and v respectively.

At the lowest point, tension = $T_L = mg + \frac{mu^2}{\ell}$ ...(i)

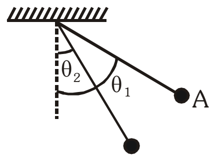
At the highest point, tension = $T_H = \frac{mv^2}{\ell} - mg$ . ..(ii)

By conservation of mechanical energy, $\frac{mu^2}{2} - \frac{mv^2}{2} = mg(2\ell)$ $\Rightarrow$ $u^2 = v^2 + 4g\ell$

Substituting this in eqn (i) $T_L = mg + \frac{m[v^2 + 4g\ell]}{\ell}$ ...(iii)

$\therefore$ From eq^n. (ii) & (iii) $T_L - T_H = 6mg$

Illustration 17.

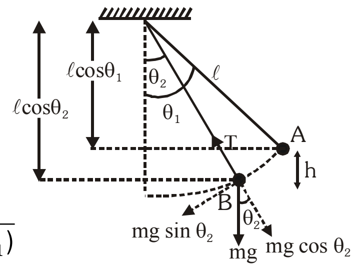


A particle of mass m is connected to a light inextensible string of length ℓ such that it behaves as a simple pendulum. Now the string is pulled to point A making an angle θ_1 with the vertical and is released then obtain expressions for the :

(i) speed of the particle and **[AIPMT (Mains) 2008]**

(ii) the tension in the string when it makes an angle θ_2 with the vertical.

Solution



(i) $h = \ell(\cos\theta_2 - \cos\theta_1)$

Applying conservation of mechanical energy between points A & B

$\frac{1}{2}mv^2 = mgh \Rightarrow v = \sqrt{2gh} = \sqrt{2g\ell(\cos\theta_2 - \cos\theta_1)}$

(ii) At position B, $T - mg\cos\theta_2 = \frac{mv^2}{\ell}$ where $v = \sqrt{2g\ell(\cos\theta_2 - \cos\theta_1)}$

$$\Rightarrow T = mg\cos\theta_2 + \frac{m}{\ell}[2g\ell(\cos\theta_2 - \cos\theta_1)] = mg(3\cos\theta_2 - 2\cos\theta_1).$$

BEGINNER'S BOX-2

1. A particle of mass m_1 is fastened to one end of a string and another one of mass m_2 to the middle point; the other end of the string being fastened to a fixed point on a smooth horizontal table. The particles are then projected, so that the two portions of the string are always in the same straight line and describe horizontal circles. Find the ratio of the tensions in the two parts of the string.

2. A road is 8 m wide. Its average radius of curvature is 40 m. The outer edge is above the lower edge by a distance of 1.28 m. Find the velocity of vehicle for which the road is most suited ? ($g = 10\ m/s^2$)

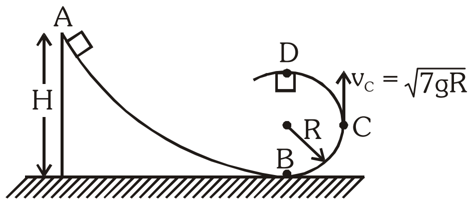
3. A stone of mass 1 kg tied to a light string of length ℓ = 10 m is whirling in a circular path in the vertical plane. If the ratio of the maximum to minimum tensions in the string is 3, find the speeds of the stone at the lowest and highest points.

4. Calculate the following for the situation shown :-

(a) Speed at D

(b) Normal reaction at D

(c) Height H



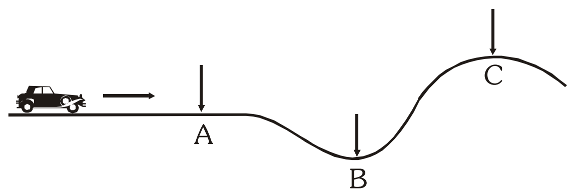
5. A car is moving along a hilly road as shown (side view). The coefficient of static friction between the tyres and the pavement is constant and the car maintains a steady speed. If at one of the points shown the driver applies brakes as hard as possible without making the tyres slip, the magnitude of the frictional force immediately after the brakes are applied will be maximum if the car was at :-

(A) point A

(B) point B

(C) point C

(D) friction force same for positions A, B and C



6. A stone weighing 0.5 kg tied to a rope of length 0.5 m revolves along a circular path in a vertical plane. The tension of the rope at the bottom point of the circle is 45 newton. To what height will the stone rise if the rope breaks at the moment when the velocity is directed upwards? ($g=10$ m/s^2)

ANSWER KEYS

BEGINNER'S BOX-1

1. (C)
2. (D)
3. 6 m/s^2 towards the centre.
4. 1 : 1
5. (C)
6. (C)
7. (A)
8. $-3\hat{k}$ rad/s^2 , $-2\hat{k}$ rad/s

BEGINNER'S BOX-2

1. $\frac{2m_1}{m_2+2m_1}$
2. 8 m/s
3. $v_{lowest} = 20\sqrt{2}$ m/s; $v_{highest} = 20$ m/s
4. (a) $\sqrt{5gR}$ (b) 4mg (c) $\frac{9}{2}R$
5. (B)
6. 1.5 m

EXERCISE-I (Conceptual Questions)

KINEMATICS OF CIRCULAR MOTION

1. A particle of mass 'm' describes a circle of radius (r). The centripetal acceleration of the particle is $\frac{4}{r^2}$. The momentum of the particle :–

(1) $\frac{2m}{r}$ (2) $\frac{2m}{\sqrt{r}}$

(3) $\frac{4m}{r}$ (4) $\frac{4m}{\sqrt{r}}$

2. A particle is moving around a circular path with uniform angular speed (ω). The radius of the circular path is (r). The acceleration of the particle is :–

(1) $\frac{\omega^2}{r}$ (2) $\frac{\omega}{r}$

(3) $v\omega$ (4) vr

3. A car moves on a circular road, describing equal angles about the centre in equal intervals of times. Which of the statements about the velocity of car is true :–

(1) velocity is constant

(2) magnitude of velocity is constant but the direction of velocity change

(3) both magnitude and direction of velocity change

(4) velocity is directed towards the centre of circle

4. An insect trapped in a circular groove of radius 12 cm moves along the groove steadily and completes 7 revolutions in 100 s. What is the linear speed of the motion :–

(1) 2.3 cm/s (2) 5.3 cm/s

(3) 0.44 cm/s (4) none of these

5. A particle moves in a circle of the radius 25 cm at two revolutions per second. The acceleration of the particle in m/sec^2 is :–

(1) π^2 (2) $8\pi^2$

(3) $4\pi^2$ (4) $2\pi^2$

6. A particle moves in a circle describing equal angle in equal times, its velocity vector :–

(1) remains constant

(2) change in magnitude

(3) change in direction

(4) changes in magnitude and direction

7. A mass of 2 kg is whirled in a horizontal circle by means of a string at an initial speed of 5 r.p.m. Keeping the radius constant the tension in the string is doubled. The new speed is nearly :–

(1) 7 r.p.m. (2) 14 r.p.m.

(3) 10 r.p.m. (4) 20 r.p.m.

8. A particle moving along a circular path. The angular velocity, linear velocity, angular acceleration and centripetal acceleration of the particle at any instant respectively are $\vec{\omega}$, $\vec{v}$, $\vec{\alpha}$, $\vec{a}_c$. Which of the following relation is/are correct :–

(a) $\vec{\omega} \perp \vec{v}$ (b) $\vec{\omega} \perp \vec{\alpha}$ (c) $\vec{v} \perp \vec{a}_c$ (d) $\vec{\omega} \perp \vec{a}_c$

(1) a,b,d (2) b,c,d (3) a,b,c (4) a,c,d

9. A particle is acted upon by a force of constant magnitude which is always perpendicular to the velocity of the particle. The motion of the particle takes place in a plane. It follows, that :–

(1) its velocity is constant

(2) its K.E. is constant

(3) its acceleration is constant

(4) it moves in a straight line

10. If the equation for the displacement of a particle moving on a circular path is given by $(\theta) = 2t^3 + 0.5$, where θ is in radians and t in seconds, then the angular velocity of the particle after 2 s from its start is :–

(1) 8 rad/s (2) 12 rad/s

(3) 24 rad/s (4) 36 rad/s

11. A sphere of mass m is tied to end of string of length ℓ and rotated through the other end along a horizontal circular path with speed v. The work done in full horizontal circle is :-

(1) 0 (2) $\left(\frac{mv^2}{\ell}\right).2\pi\ell$

(3) mg.$2\pi\ell$ (4) $\left(\frac{mv^2}{\ell}\right).\ell$

12. A body moves with constant angular velocity on a circle. Magnitude of angular acceleration :-

(1) $r\omega^2$ (2) Constant

(3) Zero (4) None of the above

13. A particle of mass (m) revolving in horizontal circle of radius (R) with uniform speed v. When particle goes from one end to other end of diameter, then :-

(1) K.E. changes by $\frac{1}{2}mv^2$

(2) K.E. change by mv^2

(3) no change in momentum

(4) change in momentum is 2 mv

14. A stone is tied to one end of string 50 cm long and is whirled in a horizontal circle with constant speed. If the stone makes 10 revolutions in 20 s, then what is the magnitude of acceleration of the stone :-

(1) 493 cm/s^2 (2) 720 cm/s^2

(3) 860 cm/s^2 (4) 990 cm/s^2

15. For a particle in a non-uniform accelerated circular motion :–

(1) velocity is radial and acceleration is transverse only

(2) velocity is transverse and acceleration is radial only

(3) velocity is radial and acceleration has both radial and transverse components

(4) velocity is transverse and acceleration has both radial and transverse components

16. The angular velocity of a particle rotating in a circular orbit 100 times per minute is

(1) 1.66 rad / s (2) 10.47 rad / s

(3) 10.47 degree / s (4) 60 degree / s

17. Two particles having mass 'M' and 'm' are moving in a circular path having radius R and r. If their time period are same then the ratio of angular velocity will be :–

(1) $\frac{r}{R}$ (2) $\frac{R}{r}$ (3) 1 (4) $\sqrt{\frac{R}{r}}$

18. Angular velocity of minute hand of a clock is :–

(1) $\frac{\pi}{30}$ rad/s (2) 8π rad/s

(3) $\frac{2\pi}{1800}$ rad/s (4) $\frac{\pi}{1800}$ rad/s

19. A car moving with speed 30 m/s on a circular path of radius 500 m. Its speed is increasing at the rate of 2m/s^2. The acceleration of the car is :–

(1) 9.8 m/s^2 (2) 1.8 m/s^2

(3) 2 m/s^2 (4) 2.7 m/s^2

20. If a particle is rotating uniformly in a horizontal circle, then –

(1) no force is acting on particle

(2) velocity of particle is constant

(3) particle has no acceleration

(4) no work is done

21. A body of mass 1 kg tied to one end of string is revolved in a horizontal circle of radius 0.1 m with a speed of 3 revolution/sec, assuming the effect of gravity is negligible, then linear velocity, acceleration and tension in the string will be :–

(1) 1.88 m/s, 35.5 m/s^2, 35.5 N

(2) 2.88 m/s, 45.5 m/s^2, 45.5 N

(3) 3.88 m/s, 55.5 m/s^2, 55.5 N

(4) None of these

22. A particle moves along a circle of radius ($\frac{20}{\pi}$) m with constant tangential acceleration. If the velocity of the particle is 80 m/s at the end of the second revolution after motion has begun, the tangential acceleration is :–

(1) 40 m/s^2 (2) 640 π m/s^2

(3) 160 π m/s^2 (4) 40 π m/s^2

23. The angular velocity of a wheel is 70 rad/s. If the radius of the wheel is 0.5 m, then linear velocity of the wheel is :–

(1) 70 m/s (2) 35 m/s

(3) 30 m/s (4) 20 m/s

24. A stone tied to the end of a string of 1m long is whirled in a horizontal circle with a constant speed. If the stone makes 22 revolution in 44 seconds, what is the magnitude and direction of acceleration of the stone :–

(1) π^2 m/s^2 and direction along the tangent to the circle.

(2) π^2 m/s^2 and direction along the radius towards the centre.

(3) $\frac{\pi^2}{4}$ m/s^2 and direction along the radius towards the centre.

(4) π^2 m/s^2 and direction along the radius away from the centre.

25. A fly wheel rotating at 600 rev/min is brought under uniform deceleration and stopped after 2 minutes, then what is angular deceleration in rad/sec^2 ?

(1) $\frac{\pi}{6}$ (2) 10 π (3) $\frac{1}{12}$ (4) 300

26. The linear and angular acceleration of a particle are 10 m/s^2 and 5 rad/s^2 respectively. It will be at a distance from the axis of rotation.

(1) 50 m (2) $\frac{1}{2}$ m (3) 1 m (4) 2 m

DYNAMICS OF HORIZONTAL CIRCULAR MOTION

27. The angular acceleration of particle moving along a circular path with uniform speed :–

(1) uniform but non zero

(2) zero

(3) variable

(4) as can not be predicted from given information

28. If the speed and radius both are trippled for a body moving on a circular path, then the new centripetal force will be :–

(1) Doubled of previous value

(2) Equal to previous value

(3) Triple of previous value

(4) One third of previous value

29. When a body moves with a constant speed along a circle :–

(1) no acceleration is present in the body

(2) no force acts on the body

(3) its velocity remains constant

(4) no work gets done on it

30. A pendulum is suspended from the roof of a rail road car. When the car is moving on a circular track the pendulum inclines :

(1) Forward

(2) Backward

(3) Towards the centre of the path

(4) Away from the centre of the path

31. A string of length 0.1 m cannot bear a tension more than 100N. It is tied to a body of mass 100g and rotated in a horizontal circle. The maximum angular velocity can be -

(1) 100 rad/s (2) 1000 rad/s

(3) 10000 rad/s (4) 0.1 rad/s

32. The radius of the circular path of a particle is doubled but its frequency of rotation is kept constant. If the initial centripetal force be F, then the final value of centripetal force will be :–

(1) F (2) $\frac{F}{2}$ (3) 4F (4) 2F

33. A 0.5 kg ball moves in a circle of radius 0.4 m at a speed of 4 m/s. The centripetal force on the ball is :–

(1) 10N (2) 20N (3) 40N (4) 80N

34. A body is revolving with a constant speed along a circle. If its direction of motion is reversed but the speed remains the same then :–

(a) the centripetal force will not suffer any change in magnitude

(b) the centripetal force will have its direction reversed

(c) the centripetal force will not suffer any change in direction

(d) the centripetal force is doubled

(1) a,b (2) b,c (3) c,d (4) a, c

35. a_r and a_t represent radial and tangential acceleration. The motion of a particle will be uniform circular motion, if :–

(1) $a_r = 0$ and $a_t = 0$ (2) $a_r = 0$ but $a_t \neq 0$

(3) $a_r \neq 0$ but $a_t = 0$ (4) $a_r \neq 0$ and $a_t \neq 0$

36. In uniform circular motion, the velocity vector and acceleration vector are

(1) Perpendicular to each other

(2) Same direction

(3) Opposite direction

(4) Not related to each other

37. A string of length 10 cm breaks if its tension exceeds 10 newton. A stone of mass 250 g tied to this string, is rotated in a horizontal circle. The maximum angular velocity of rotation can be :-

(1) 20 rad/s (2) 40 rad/s

(3) 100 rad/s (4) 200 rad/s

38. The earth ($M_e = 6 \times 10^{24}$ kg) is revolving round the sun in an orbit of radius (1.5×10^8) km with angular velocity of (2×10^{-7}) rad/s. The force (in newton) exerted on the earth by the sun will be :-

(1) 36×10^{21} (2) 16×10^{24}

(3) 25×10^{16} (4) Zero

39. A 500 kg car takes a round turn of radius 50 m with a velocity of 36 km/hr. The centripetal force is :-

(1) 250 N (2) 1000N (3) 750N (4) 1200 N

40. A motor cycle driver doubles its velocity when he is taking a turn. The force exerted towards the centre will become :-

(1) double (2) half

(3) 4 times (4) $\frac{1}{4}$ times

41. The force required to keep a body in uniform circular motion is :-

(1) Centripetal force (2) Centrifugal force

(3) Resistance (4) None of the above

BANKING OF TRACKS

42. A car moving on a horizontal road may be thrown out of the road in taking a turn :–

(1) by the gravitational force

(2) due to lack of proper centripetal force

(3) due to rolling friction between the tyres and the Road

(4) due to reaction of the road

43. Radius of the curved road on national highway is R. Width of the road is b. The outer edge of the road is raised by h with respect to inner edge so that a car with velocity v can pass safely over it. The value of h is :–

(1) $\frac{v^2b}{Rg}$ (2) $\frac{v}{Rgb}$ (3) $\frac{v^2R}{bg}$ (4) $\frac{v^2b}{R}$

44. A boy holds a pendulum in his hand while standing at the edge of a circular platform of radius r rotating at an angular speed ω. The pendulum will hang at an angle θ with the vertical so that :–
(Neglect length of pendulum)

(1) $\tan\theta = 0$ (2) $\tan\theta = \frac{\omega^2 r^2}{g}$

(3) $\tan\theta = \frac{r\omega^2}{g}$ (4) $\tan\theta = \frac{g}{\omega^2 r}$

VERTICAL CIRCULAR MOTION

45. Let 'θ' denote the angular displacement of a simple pendulum oscillating in a vertical plane. If the mass of the bob is (m), then the tension in string is mg cosθ :–

(1) always

(2) never

(3) at the extreme positions

(4) at the mean position

46. A pendulum bob has a speed 3 m/s while passing through its lowest position, length of the pendulum is 0.5 m then its speed when it makes an angle of 60° with the vertical is :–(g = 10 m/s²)

(1) 2 m/s (2) 1 m/s (3) 4 m/s (4) 3 m/s

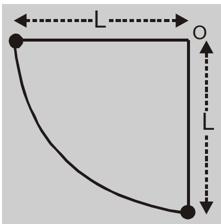
47. The mass of the bob of a simple pendulum of length L is m. If the bob is left from its horizontal position then the speed of the bob and the tension in the thread in the lowest position of the bob will be respectively :–

(1) $\sqrt{2gL}$ and 3 mg

(2) 3 mg and $\sqrt{2gL}$

(3) 2 mg and $\sqrt{2gL}$

(4) 2 gL and 3 mg



48. A stone of mass 1 kg is tied to the end of a string of 1 m length. It is whirled in a vertical circle. If the velocity of the stone at the top be 4 m/s. What is the tension in the string (at that instant) ?

(1) 6 N (2) 16 N (3) 5 N (4) 10 N

49. In a vertical circle of radius (r), at what point in its path a particle may have tension equal to zero :–

(1) highest point

(2) lowest point

(3) at any point

(4) at a point horizontal from the centre of radius

50. A stone attached to one end of a string is whirled in a vertical circle. The tension in the string is maximum when :–

(1) the string is horizontal

(2) the string is vertical with the stone at highest position

(3) the string is vertical with the stone at the lowest position

(4) the string makes an angle of 45° with the vertical

51. A weightless thread can withstand tension upto 30 N. A stone of mass 0.5 kg is tied to it and is revolved in a circular path of radius 2m in a vertical plane. If g = 10 m/s², then the maximum angular velocity of the stone can be :–

(1) 5 rad/s (2) $\sqrt{30}$ rad/s

(3) $\sqrt{60}$ rad/s (4) 10 rad/s

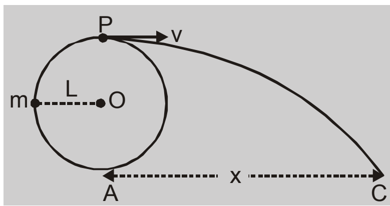
52. A body tied to a string of length L is revolved in a vertical circle with minimum velocity, when the body reaches the upper most point the string breaks and the body moves under the influence of the gravitational field of earth along a parabolic path. The horizontal range AC of the body will be :–

(1) x = L

(2) x = 2L

(3) $x = 2\sqrt{2}L$

(4) $x = \sqrt{2}L$



53. A particle is moving in a vertical circle the tension in the string when passing through two position at angle 30° and 60° from vertical from lowest position are T_1 and T_2 respectively then :–

(1) $T_1 = T_2$ (2) $T_1 > T_2$

(3) $T_1 < T_2$ (4) $T_1 \geq T_2$

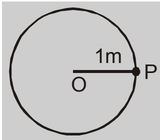
54. A body crosses the topmost point of a vertical circle with critical speed. What will be its centripetal acceleration when the string is horizontal :–

(1) g (2) 2g (3) 3g (4) 6g

55. Stone tied at one end of light string is whirled round a vertical circle. If the difference between the maximum and minimum tension experienced by the string wire is 2 kg wt, then the mass of the stone must be :–

(1) 1 kg (2) 6 kg (3) 1/3 kg (4) 2 kg

56. A mass tied to a string moves in a vertical circle with a uniform speed of 5 m/s as shown. At the point P the string breaks. The mass will reach a height above P of nearly ($g = 10\ m/s^2$) :–



(1) 1 m

(2) 0.5 m

(3) 1.75 m

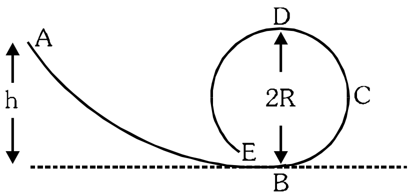
(4) 1.25 m

57. If the overbridge is concave instead of being convex, then the thrust on the road at the lowest position will be :-

(1) $mg + \frac{mv^2}{r}$ (2) $mg - \frac{mv^2}{r}$

(3) $\frac{m^2v^2g}{r}$ (4) $\frac{v^2g}{r}$

58. A frictionless track ABCDE ends in a circular loop of radius R. A body slides down the track from point A which is at a height h = 5 cm. Maximum value of R for the body to successfully complete the loop is :–



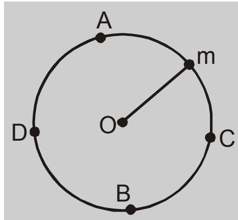
(1) 5 cm.

(2) 2 cm.

(3) $\frac{10}{3}$ cm.

(4) $\frac{15}{4}$ cm.

59. A particle of mass m is performing vertical circulamotion (see figure). If the average speed of the particle is increased, then at which point maximum breaking possibility of the string :–



(1) A

(2) B

(3) C

(4) D

60. A fighter plane is moving in a vertical circle of radius 'r'. Its minimum velocity at the highest point of the circle will be :–

(1) $\sqrt{3gr}$ (2) $\sqrt{2gr}$

(3) $\sqrt{gr}$ (4) $\sqrt{\frac{gr}{2}}$

61. A stone of mass 0.2 kg is tied to one end of a thread of length 0.1 m whirled in a vertical circle. When the stone is at the lowest point of circle, tension in thread is 52N, then velocity of the stone will be :–

(1) 4 m/s (2) 5 m/s (3) 6 m/s (4) 7 m/s

62. A suspended simple pendulum of length ℓ is making an angle θ with the vertical. On releasing, its velocity at lowest point will be :-

(1) $\sqrt{2g\ell(1+\cos\theta)}$ (2) $\sqrt{2g\ell\sin\theta}$

(3) $\sqrt{2g\ell(1-\cos\theta)}$ (4) $\sqrt{2g\ell}$

EXERCISE-I (Conceptual Questions) ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	**2**	**3**	**2**	**2**	**3**	**3**	**1**	**4**	**2**	**3**	**1**	**3**	**4**	**1**	**4**
Que.	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Ans.	**2**	**3**	**4**	**4**	**4**	**1**	**1**	**2**	**2**	**1**	**4**	**2**	**3**	**4**	**4**
Que.	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
Ans.	**1**	**4**	**2**	**4**	**3**	**1**	**1**	**1**	**2**	**3**	**1**	**2**	**1**	**3**	**3**
Que.	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
Ans.	**1**	**1**	**1**	**1**	**3**	**1**	**2**	**2**	**3**	**3**	**4**	**1**	**2**	**2**	**3**
Que.	61	62													
Ans.	**2**	**3**													

EXERCISE-II (Assertion & Reason)

Directions for Assertion & Reason questions

These questions consist of two statements each, printed as Assertion and Reason. While answering these Questions you are required to choose any one of the following four responses.

(A) If both Assertion & Reason are True & the Reason is a correct explanation of the Assertion.

(B) If both Assertion & Reason are True but Reason is not a correct explanation of the Assertion.

(C) If Assertion is True but the Reason is False.

(D) If both Assertion & Reason are false.

1. ***Assertion :*** In uniform circular motion, the linear speed and angular speed of the body are constant.
Reason : A body can move on a circular path without having acceleration.
(1) A (2) B (3) C (4) D

2. ***Assertion :*** The resultant acceleration of an object in circular motion is towards the centre if the speed is constant.
Reason : A vector is necessarily changed if it is rotated through an angle.
(1) A (2) B (3) C (4) D

3. ***Assertion :*** Work done by the centripetal force in moving a body along a circle is always zero.
Reason : In circular motion the displacement of the body is along the force.
(1) A (2) B (3) C (4) D

4. ***Assertion :*** Centripetal and centrifugal forces cancel each other.
Reason : This is because they are always equal and opposite.
(1) A (2) B (3) C (4) D

5. ***Assertion :*** Work done by centripetal force is zero.
Reason : Centripetal force acts perpendicular to the displacement.
(1) A (2) B (3) C (4) D

6. ***Assertion :*** When a particle moves in a circle with a uniform speed its acceleration is constant but the velocity changes.
Reason : Angular displacement is not an axial vector.
(1) A (2) B (3) C (4) D

7. ***Assertion :*** If a particle moves along circular path with constant speed then acceleration must be present.
Reason : If a particle moves with variable velocity then acceleration must be present.
(1) A (2) B (3) C (4) D

8. ***Assertion :*** In uniform circular motion speed of particle must be constant.
Reason : In uniform circular motion no force or acceleration is acting on particle acting parallel or in anti parallel to the direction of velocity
(1) A (2) B (3) C (4) D

9. ***Assertion :*** When the direction of motion of a particle moving in a circular path is reversed the direction of radial acceleration still remains the same (at the given point).
Reason : For a particle revolving on circular path in any direction such as clockwise or anticlockwise, the direction of radial acceleration is always towards the centre of the circular path.
(1) A (2) B (3) C (4) D

10. ***Assertion :*** For uniform circular motion it is necessary that the speed of the particle is constant.
Reason : There is no tangential force or tangential acceleration acting on particle in uniform circular motion.
(1) A (2) B (3) C (4) D

11. ***Assertion :*** Acceleration of the particle in uniform circular motion remains constant.
Reason : Velocity of the particle doesn't change in circular motion.
(1) A (2) B (3) C (4) D

12. ***Assertion :*** In the inertial frame centrifugal force can't appear.
Reason : In the uniform circular motion centripetal force will counter balance to the centrifugal force.
(1) A (2) B (3) C (4) D

13. ***Assertion :*** During a turn, the value of centripetal force should be less than the limiting frictional force.
Reason : The centripetal force is provided by the frictional force between the tyres and the road.
(1) A (2) B (3) C (4) D

14. ***Assertion :*** A car moving on a horizontal road may be thrown out of the road in taking a turn due to lack of proper centripetal force.
Reason : If a particle moves in a circle, describing equal angles in equal intervals of time. Then the velocity vector changes its magnitude.
(1) A (2) B (3) C (4) D

15. ***Assertion :-*** When a car turns around a circular path it is acted upon by a centripetal force.
Reason :- The friction acting on the wheels of car provides necessary centripetal force.
(1) A (2) B (3) C (4) D

EXERCISE-II (Assertion & Reason) ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	3	3	3	4	1	4	1	1	1	1	4	3	1	3	1